Laplace assumed such law of increase of density from surface to center as would make the differential equation for the ellipticities of the strata

$$\frac{d^2}{da^2}\Big[\varphi(a)e\Big] = \frac{6}{a^2}\,\varphi(a)e + 3a^2 e\frac{d\rho}{da} \qquad (1)$$

integrable.

By assuming that the last term of the second member of equation (1) is a function of a, Pratt, in his "Figure of the Earth," deduced the result

$$\rho = Q\frac{\sin qa}{a},$$

in which $\rho =$ the density, Q and q are constants, and $a =$ radius of shell considered as spherical.

The constants q and Q are found from the conditions that the density at the surface is 2.75, and the mean density of the Earth is twice that of the surface.

By means of the densities thus found, Pratt deduces the masses of the nucleus, and four concentric shells, the radius of the nucleus and the thickness of each shell being equal to one fifth of the Earth's radius.

The masses in order from the surface are, 0.3372, 0.3257, 0.2237, 0.0981, 0.0153, the mass of the Earth being unity.

From these values the masses of the parts whose radii are R, $\frac{4}{5}R$, $\frac{3}{5}R$, $\frac{2}{5}R$ and $\frac{1}{5}R$ are found to be 1.0000, 0.6628, 0.3371, 0.1134, 0.0153.

Taking the formula for attraction at the surface, each mass considered a sphere, $A = m \div R^2$, we get in order from the surface, $\frac{1}{25}(1.0000)$, $\frac{1}{16}(0.6628)$, $\frac{1}{9}(0.3371)$, $\frac{1}{4}(0.1134)$, 0.0153, or, 0.0400, 0.0414, 0.03746, 0.0284, 0.0153.

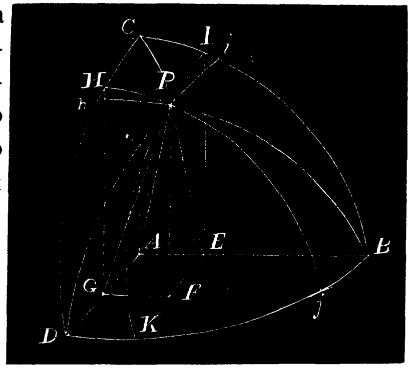
If this is the true law, the force of attraction increases from the surface till we reach a point perhaps one-third distant from surface to center, from that point the force diminishes and becomes zero at the center.

SOLUTION OF THE PROBLEM PROPOSED BY MR. HARVILL AT THE CLOSE OF HIS SOLUTION ON PAGE 56.

BY THE EDITOR.

LET A represent the center of the sphere, radius r, BCD a tri-rectangular spherical triangle, and $AEFGHPCI$, the solid removed from this portion of the sphere in making the first hole.

Draw BPh, DPi and CP, arcs of great circles respectively through BP, DP and CP, and draw HPj, IPk, small circles thro' P with poles at B, D.



The solid $AEFGHPIC$ consists of a square pyramid, $AEFGP$ plus two similar and equal triangular spherical pyramids, $CBPA$ and $CDPA$, *minus* two equal sectors $BPIE$ and $DPHG$, of two equal spherical segments whose height is $BE = DG$.

Putting $2x$ for the width of the chisel, we have $AE = EF = x$, and consequently FP, the altitude of the square pyramid $= \surd(r^2 - 2x^2)$; hence the solidity of the pyramid $AEFGP = \frac{1}{3}x^2\surd(r^2 - 2x^2)$.

The base of the spherical pyramid $BCPA$, is the area of the spherical triangle $BCP = S$.

To find S, we have $\angle C = 45°$, $\angle B = \sin^{-1}[x \div \surd(r^2 - x^2)]$, whence we find $\angle P = \sin^{-1}[\surd(r^2 - 2x^2) \div \surd(2r^2 - 2x^2)] + 90°$; therefore the sum of the angles of the spherical triangle BCP is

$$\sin^{-1}\left[\frac{r+2x}{(r+x).\surd 2}\right] + 135°;$$

$$\therefore S = \left\{\sin^{-1}\left[\frac{r+2x}{(r+x).\surd 2}\right] - 45°\right\}\frac{S'}{90°},$$

where S' is the area of the spherical trian. $ABC = \frac{1}{8}$ the surface of the sphere $= \frac{1}{2}\pi r^2$. Hence the solidity of the two equal spherical pyramids is

$$\frac{\frac{1}{3}\pi r^3}{90°}\left\{\sin^{-1}\left[\frac{r+2x}{(r+x).\surd 2}\right] - 45°\right\}.$$

The solidity of the sectors to be subt. is $\frac{\frac{1}{3}(4r^3 - 6r^2x + 2x^3)}{360°}.\sin^{-1}\frac{x}{\surd(r^2 - x^2)}$; hence $\frac{1}{8}$ the solidity removed in cutting the first hole through the sphere is

$$\frac{x^2\surd(r^2 - 2x^2)}{3} + \frac{\frac{1}{3}\pi r^3}{90°}\left\{\sin^{-1}\left[\frac{r+2x}{(r+x).\surd 2}\right] - 45°\right\}$$
$$- \frac{\frac{1}{3}(4r^3 - 6r^2x + 2x^3)}{360°}.\sin^{-1}\frac{x}{\surd(r^2 - x^2)},$$

and consequently the part of the sphere removed in making the two holes will be 16 times the above $-8x^3$, the part common to both holes. Therefore

$$\frac{16x^2\surd(r^2 - 2x^2)}{3} + \frac{\frac{16}{3}\pi r^3}{90°}\left\{\sin^{-1}\left[\frac{r+2x}{(r+x).\surd 2}\right] - 45°\right\} - 8x^3$$
$$- \frac{\frac{16}{3}(4r^3 - 6r^2x + 2x^3)}{360°}.\sin^{-1}\frac{x}{\surd(r^2 - x^2)} = \frac{2\pi r^3}{3}.$$

By solving this transcendental equation the value of x will be found.